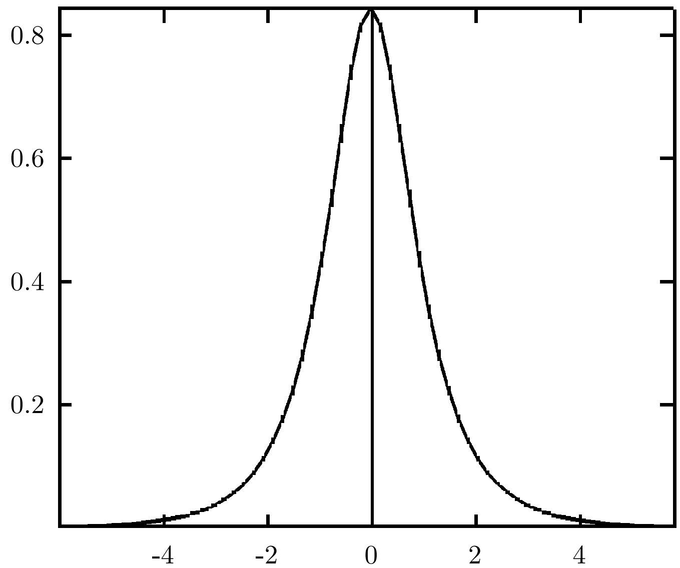


FIG. 1. Plot of the function $\rho(\zeta_x, 0) = \rho(0, \zeta_y)$.

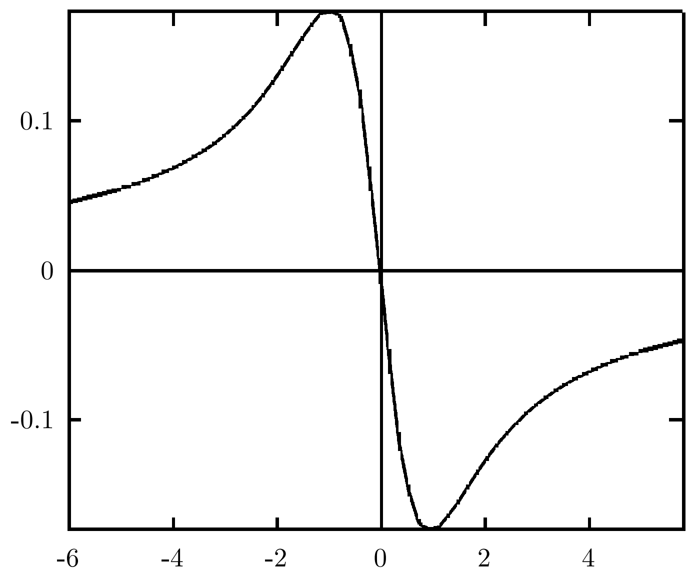


FIG. 2. Plot of the function $u(\zeta_x, 0)$.

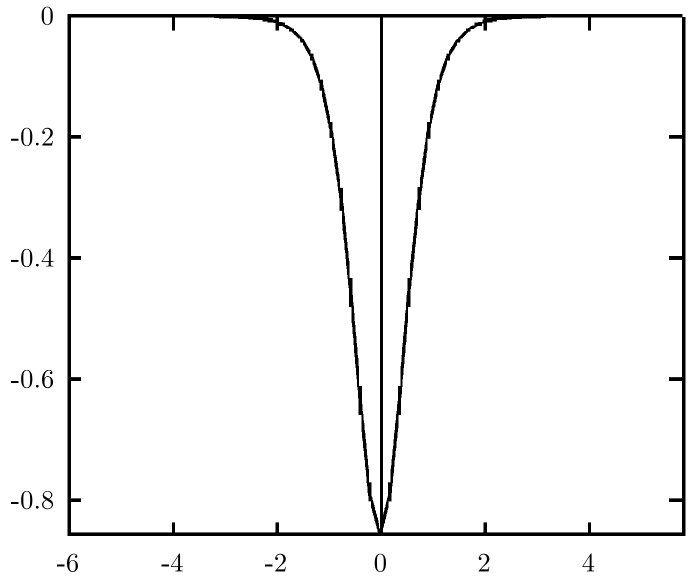
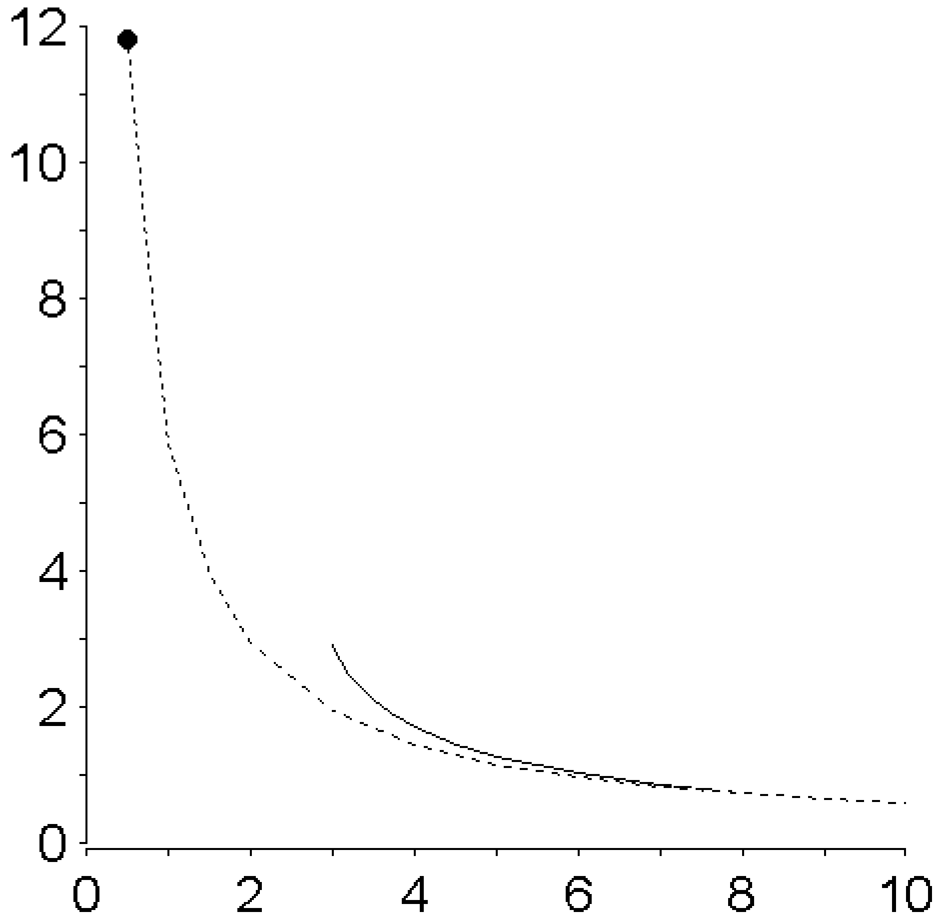


FIG. 3. Plot of $w(\zeta_x, 0) = w(0, \zeta_y)$.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COLLAPSE VERSUS TURBULENCE[1]

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ABSTRACT

We study the solutions of the equations of motion in the gauged (2+1)-dimensional nonlinear Schrödinger model. The contribution of Chern-Simons gauge fields leads to a significant decrease of the critical power of self-focusing. We also show that at appropriate boundary conditions in the considered model there exists a regime of turbulent motions of hydrodynamic type.

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1. Introduction

The nonlinear Schrödinger equation (NSE) is one of the basic models for nonlinear waves. The conventional field for NSE application is nonlinear optics [1, 2] wherein it describes propagation of wave beams in dispersive nonlinear media. The NSE appears also when one considers various nonlinear waves in hydrodynamics and plasma physics [3]. A most important application area is the problem of detailed description of collapsing field distributions for the NSE with local cubic nonlinearity [4, 5]. In the case of the opposite sign of nonlinearity the NSE is used as the basic model [6] of the low-dimension field theory to describe vortices in the problem of Bose condensation [7].

Recent interest in problems with NSE solving in spatially $2D$ systems was associated with the account of the specific character of $2+1D$ systems, which is manifested in equipping the NSE with a gauge field by replacing the usual derivatives by covariant ones. By that, the gauge field satisfies its own equation of motion (EM) with current, determined by the NSE solution. In the infrared limit, the main contribution to the EM of the gauge field in $2+1D$ system is given by the Chern-Simons (CS) term within the system under consideration. The contribution of the gauge fields compensates in the Hamiltonian the contribution of nonlinearity at a certain relation of coupling constants. It leads to a soliton distribution of the fields found in Ref. [8]. The nature of this phenomenon can easily be understood if one takes into account that CS term breaks $\mathcal{T}$- and $\mathcal{P}$-inversion symmetry of time and space. The chosen direction of the vector in the direction perpendicular to the plane can be considered as a chosen direction of rotation in the plane leading to the appearance of efficient repulsion. When this repulsion compensates the attraction, the Hamiltonian turns out to be limited at the bottom, and the CS solitons [8] correspond to its zero value. These field distributions are solutions of the self-dual equations [9].

The results of the Refs. [8, 10, 11] stimulated a series of papers in this field. We would like to pay attention to some of them. In Ref. [12] the structures of field configurations were analyzed in full detail for a nonlinearity in NSE, which describes repulsion (in the absence of CS interaction) and takes into account the contribution of the non-zero mean value of particle number density. Considering the initial problem the authors of Ref. [13] concluded that at the most general initial condition the problems of EM for NSE with CS gauge fields correspond to the collapse regime. However, neither the spatial structure of the collapsing mode, nor the critical power for it (i.e. the number of particles in the mode) have been analyzed in this paper. The problem of exact integration of the model under consideration was analyzed in Ref. [14]. The main result obtained is that the system cannot be integrated exactly excluding the following two cases: self-dual limit [8], and the situation when the $2+1D$ equations can be reduced to the $1+1D$ equations.

An additional but still very important result of that paper was that the solitons of the gauged nonlinear Schrödinger equation (GNSE) have movable singularities on some curves in the two-dimensional plane. Detailed study of topological defects in low-dimensional systems have always been a key point for understanding dynamics of field distributions. The problem of so-called semi-local topological defects in the Chern-Simons-Higgs model was considered in a recent paper [15].

The distribution of complex-valued functions $\Psi(x, y, t)$ are defined on the manifold $\mathcal{M}$ which is multiconnected in spatial $2D$ systems. Therefore the fundamental homotopy group $\pi_1(\mathcal{M})$ determining analytical properties of function Ψ coincides with the braid group. There are actually several equivalent ways to reflect this fact in the theory. One of them is the Lagrangian approach that includes the effect of CS gauge fields into consideration. The CS term codes the existence and specific character of $2D$ point peculiarities contained in the Aharonov-Bohm gauge potentials within the long-wave description. One usually speaks of the long-range interaction represented by means of the CS gauge field as a statistical interaction between different field configurations. The different representations induce the different forms of field distribution. There is the so-called anyon representation [16, 17], when the gauge field in the explicit form is excluded from the Hamiltonian of the model thus providing the representation of "non-interacting" (by means of the gauge field) configurations of field $\Psi(x, y, t)$. However, in this case the gauge field is proved to be included into the phase of function $\Psi(x, y, t)$ that contains a cut in the complex plane, which provides multivaluedness of the function. The cut describes a string attached to the point defect. Thus, the so-called non-local topological defect separates sheets in the multi-sheet coverage of the $2D$ basic space. It is well known [16, 17] that the representation with explicit presence of gauge fields in the Lagrangian has a different form depending on the parity of the permutation group representation. The approach, based on the representation with fractional statistics of field configurations, is identical to the dynamic approach, when we are interested in the influence of a "statistical" gauge field.

The existence of a gauge interaction has an exclusively topological character and is not associated with the quantum theory. This interaction, as a rule, was not taken into account when studying the classical dynamics of nonlinear models with a complex field in spatial $2D$ systems. Topological peculiarities certainly put additional restrictions to the quantization procedure in these systems [17]. The role of the CS gauge interaction in this case is to take into account *the vortex part of phase dynamics*, which was not usually considered in classical systems when the $2 + 1D$ NSE model was used.

The purpose of this paper is the study of EM in the $2 + 1D$ GNSE model. The main attention is given to the investigation of the structure of collapsing distribution of the

fields. Specifically we find, by means of numerical integration of the EM, the dependence of the critical power and efficient width of the zero-energy mode on coefficient k before the CS term. The limit $k \to \infty$, when interaction with the gauge field is negligible, may be used as a test. In this case the known values of the power and the width are restored.

If the phase of the field $\Psi(x, y, t)$ describes the longitudinal part in the gauge potential *completely*, evolution of field configurations is determined *only by temporal dependence of the gauge field*. We show that in this case the equations for the gauge field coincide with the EM of an ideal fluid. The effects of the manifestation of gauge field in classic systems with nontrivial topology, including the swimming motion at low Reynolds number within the $2+1D$ hydrodynamics are well known [18]. A new feature is the fact that the basis for the 2D turbulence basing from the Euler equation in this case is chaotic dynamics of the CS gauge field. In this sense, the GNSE is a useful hydrodynamical tool [19].

One can see the following link between the dynamics of the CS fields and the problem of $2D$ turbulence. It is well known that CS action with appropriate boundary conditions is a way to classify conformal field theories [20]. The tools of the conformal field theory, in its turn, may be used [21] to study the $2D$ turbulence. We show that within the model under consideration the connection between the dynamics of CS fields and $2D$ turbulence problem may be stated beyond the application of the conformal field theory. This dependence can be represented considering the evolution of closed current lines (loops) with the stochastization of lines near the points of link of the loops. The effect of contour links in terms of this paper reflects the effect of braiding world lines of the Aharonov-Bohm point singularities with formation of knots and links after projection of world lines onto the $2D$ space. Stochastization near the contour link points within formulation of $2D$ hydrodynamics of an ideal fluid in terms of contour variables [23, 22] was discovered in Ref. [24]. Such a stochastic behavior has a universal character. It is based on existence of the braid group and is closely connected with the arbitrary character of localization along the "time" axis of the point of world lines interlacing. Because of this the index k of the linking number proves to be a hidden parameter, which is not included into the Euler equations explicitly.

The paper is organized as follows. The second section contains the EM for the CS gauge field and the GNSE for two different Ansatz corresponding to the goals of this paper. The third section is devoted to the numerical analysis of the problem. In conclusion, the results and open questions are discussed.

2. Equations of motion

We consider a system with a Lagrangian density

$$\mathcal{L} = \frac{k}{2}\varepsilon^{\alpha\beta\gamma}A_\alpha\partial_\beta A_\gamma + i\Psi^*(\partial_t + iA_0)\Psi - \frac{1}{2}\left|(\nabla - i\mathbf{A})\Psi\right|^2 + \frac{g}{2}\left|\Psi\right|^4 . \tag{1}$$

The equations of motion (EM) have the form

$$i\partial_t\Psi = -\frac{1}{2}(\nabla - i\mathbf{A})^2\Psi + A_0\Psi - g|\Psi|^2\Psi\,, \tag{2}$$

$$[\nabla\times\mathbf{A}]_\perp = -\frac{1}{k}|\Psi|^2\,, \tag{3}$$

$$\partial_t A_i + \partial_i A_0 = -\frac{1}{k}\varepsilon_{ij}j_j\,. \tag{4}$$

Here g is the coupling constant and $\mathbf{j} = Im\Psi^*(\nabla - i\mathbf{A})\Psi$ is the current density. Hamiltonian for Eq. (1) is

$$H = \frac{1}{2}\int d^2\mathbf{r}\left(|(\nabla - i\mathbf{A})\Psi|^2 - g|\Psi|^4\right)\,, \tag{5}$$

where the potential A_μ which is the auxiliary variable is expressed in terms of $|\Psi|^2$ in the following way

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{k}\int d^2\mathbf{r}'\mathbf{G}(\mathbf{r}-\mathbf{r}')|\Psi|^2(\mathbf{r}',t)\,, \tag{6}$$

$$A_0(\mathbf{r},t) = \frac{1}{k}\int d^2\mathbf{r}'\mathbf{G}(\mathbf{r}-\mathbf{r}')\mathbf{j}(\mathbf{r}',t)\,, \tag{7}$$

where the Green function $\mathbf{G}(\mathbf{r})$

$$G_i(\mathbf{r}) = \frac{1}{2\pi}\frac{\varepsilon_{ij}x_j}{r^2} \tag{8}$$

satisfies the equation

$$\nabla\times\mathbf{G}(\mathbf{r}) = -\delta^2(\mathbf{r}) \tag{9}$$

thus A_μ is the solution of Eqs. (3) and (4). Since in Hamiltonian formulation the potentials are unambiguously presented by Eqs. (6) and (7), the gauge freedom

$$A_\mu \to A_\mu - \partial_\mu\varphi\,, \tag{10}$$

$$\Psi \to e^{i\varphi}\,\Psi \tag{11}$$

is fixed. This is achieved by the Coulomb gauge $\nabla\cdot\mathbf{A} = 0$, with boundary conditions

$$\lim_{r\to\infty} r^2 A_i(\mathbf{r},t) = \frac{1}{2\pi k}\varepsilon_{ij}x_j N\,, \tag{12}$$

$$\lim_{r\to\infty} A_0(\mathbf{r},t) = 0\,. \tag{13}$$

The choice of the boundary condition (12) is associated with the necessity to satisfy the integral representation of Gauss law (3) of CS dynamics

$$\Phi = \int d^2\mathbf{r}\,[\nabla \times \mathbf{A}]_\perp = -\frac{1}{k}\int d^2\mathbf{r}|\Psi|^2\,. \tag{14}$$

Here the magnetic flux Φ and the number of particles

$$N = \int d^2\mathbf{r}|\Psi|^2 \tag{15}$$

are conserved giving the global constrain $\Phi = -\frac{1}{k}N$. In the result of Eqs. (2)-(4), there exists naturally the continuity equation

$$\partial_t|\Psi|^2 + \nabla\cdot\mathbf{j} = 0\,, \tag{16}$$

Let us use dimensionless fields and coordinates obtained by the following substitutions

$$\Psi = |k|^{3/2}\rho e^{i\varphi}, \quad A_0 = -\frac{k^2}{2}w - \partial_t\varphi, \quad A_x = -k\,u + \partial_x\varphi, \quad A_y = -k\,v + \partial_y\varphi, \tag{17}$$

$$t \to \frac{-2}{k|k|}t\,, \quad x \to \frac{x}{|k|}\,, \quad y \to \frac{y}{|k|}\,. \tag{18}$$

The EM and the continuity equation, expressed by the new real functions $\rho \equiv \rho(x,y,t)$, $u \equiv u(x,y,t)$, $v \equiv v(x,y,t)$, $w \equiv w(x,y,t)$ have the form

$$\rho_{xx} + \rho_{yy} = -2C\rho^3 - \rho w + \rho(u^2 + v^2)\,, \tag{19}$$

$$u_y - v_x = -\rho^2\,, \tag{20}$$

$$u_t - w_x = -2v\rho^2\,, \tag{21}$$

$$v_t - w_y = 2u\rho^2\,, \tag{22}$$

$$\rho^2_t = 2\left((u\rho^2)_x + (v\rho^2)_y\right) \tag{23}$$

with the parameter $C = g|k|$ and notations $u_t = \partial_t u$ etc.

In the case of usual NSE

$$i\partial_t\Psi = -\nabla^2\Psi - |\Psi|^2\Psi \tag{24}$$

substituting $\Psi = \rho e^{-i\varphi(x,y,t)}$ we have

$$\rho_{xx} + \rho_{yy} = -\rho^3 - \rho\varphi_t + \rho\left((\varphi_x)^2 + (\varphi_y)^2\right)\,, \tag{25}$$

$$\rho^2_t = 2\left((\varphi_x\rho^2)_x + (\varphi_y\rho^2)_y\right)\,. \tag{26}$$

Comparing Eqs. (25), (26) with Eqs. (19) and (23), we pay attention to the following distinctions. First, due to gauge invariance, there are no derivatives of phase φ in Eq. (19)

which exist in (25). Their role is played by gauge potentials. Therefore the evolution of the field $\rho(x,y,t)$ is defined by the time derivatives of the functions $u(x,y,t)$ and $v(x,y,t)$ in Eqs. (20), (21). Unlike Eq. (25) the fields u and v are responsible for *the transverse* dynamics of the phase of the field Ψ. *Longitudinal* dynamics of the phase is described by the zero component $w(x,y,t)$ of the gauge potential which takes the place of the function φ_t in Eq. (19). The function $w(x,y,t)$ plays the role of a Lagrange multiplier permitting to take into account the restriction (20) of the Gauss law $\Phi = -\frac{1}{k}N$ locally.

Second, the continuity equation (23) can be obtained excluding the function w from Eqs. (20) and (21) with the aid of Eq. (20). This remark is associated with the following problem. Let us assume in the Coulomb gauge $\nabla \cdot \mathbf{A} = -u_x - v_y + \Delta\varphi = 0$ that the phase φ satisfies the equation $\Delta\varphi$=0. Then the solution to the equation $u_x + v_y = 0$ may be expressed by means of the function $a(x,y,t)$ in the following way:

$$u = a_y \,, \qquad v = -a_x \,. \tag{27}$$

In this case after replacing t by $-2t$ Eqs. (20) and (23) have the form

$$a_{xx} + a_{yy} = -\rho^2 \,, \tag{28}$$

$$\rho^2_{\ t} + u\rho^2_{\ x} + v\rho^2_{\ y} = 0 \,. \tag{29}$$

The set of Eqs. (28) and (29) represents the "vorticity" form of Navier-Stokes equations (Euler equations) for two-dimensional flows of ideal incompressible fluid where the function $a(x,y,t)$ has the sense of the stream function. Note that hydrodynamic analogies have already been used for the solution of $1+1D$ NSE problem [25, 19]. However, the exact proof that dynamics of CS gauge field in the frames of GNSE model (in the particular case of Coulomb gauge with $\Delta\varphi = 0$) is equivalent to the two-dimensional EM of ideal incompressible fluid is given in this paper for the first time. The remarkable fact is that there is a close analogy between the states with the constant flux in the turbulence and CS anomaly [21] exposed by Eq. (28).

It is useful to compare the gauge invariance of the model and the used Coulomb gauge at $\Delta\varphi = 0$ with canonical transformations and with area-preserving transformations. The infinitesimal area-preserving diffeomorphism which acts in the frames of CS theory has the form

$$\xi_i \to \xi_i + \chi_i \,, \qquad \partial_i \chi_i = 0 \,, \tag{30}$$

where $\xi_i = (x,y)$, $\chi_i = (A_1, A_2)$. The general solution of the equation $\partial_i\chi_i = 0$ is the sum of two terms

$$\chi_i = \varepsilon_{ij}\partial_j a(\xi) + \sum_{k=1}^{b_1} c_k \chi_i^k \,, \tag{31}$$

where the second term describes the finite number (given by the first Betti number b_1) of harmonic forms on the two-dimensional phase space (A_x, A_y) of CS theory. Diffeomorphisms which resulted from the first term in Eq. (31) are nothing but canonical transformations [26]. In the case of phase space which is actually a torus there are also two global translations $p_k = -i\partial_k$ (for torus the first Betti number $b_1 = 2$). Just in this case the phase $\varphi(x, y, t)$, satisfying the equation $\Delta\varphi = 0$, is the linear function $\varphi = ax+by$. From the viewpoint of NSE this corresponds to the constant direction of ray propagation assigned by the vector $\mathbf{n} \sim (a, b)$. In the general case of the phase space with arbitrary topology it is invalid and the phase $\varphi(x, y, t)$ not satisfying the equation $\Delta\varphi = 0$ gives rise to the "additional" longitudinal contribution to the potentials $u(x, y, t)$ and $v(x, y, t)$.

Let us consider for example the case when the Ansatz for the field $\Psi(x, y, t)$ corresponds to the generalized lens transformation [5, 13]

$$\Psi(\mathbf{r}, t) = \frac{\Phi(\boldsymbol{\zeta}, \tau)}{g(\tau)} \exp\left(-ib(\tau)\boldsymbol{\zeta}^2/2 + i\lambda\tau\right) . \tag{32}$$

Here $\boldsymbol{\zeta} = \mathbf{r}/g(\tau)$, $\tau = \int_0^t du\,[f(u)]^{-2}$, $b(\tau) = -f_t f = -g_\tau g$. The gauge potentials transform [8], at such substitution, are as follows

$$\mathbf{A}(\mathbf{r}, t) \to [g(\tau)]^{-1}\, \mathbf{A}(\boldsymbol{\zeta}, \tau)\,, \tag{33}$$

$$A_0(\mathbf{r}, t) \to [g(\tau)]^{-2}\, [A_0(\boldsymbol{\zeta}, \tau) - b(\tau)\boldsymbol{\zeta}\mathbf{A}(\boldsymbol{\zeta}, \tau)] \tag{34}$$

the relations (6), (7), being preserved, where the function $\rho = |\Phi|$. After these transformations Eq. (2) changes its form

$$i\partial_\tau\Phi + (\beta\boldsymbol{\zeta}^2 - \lambda)\Phi = -\frac{1}{2}(\nabla - i\mathbf{A})^2\Phi + A_0\Phi - g|\Phi|^2\Phi\,, \tag{35}$$

because the function $\beta(\tau) = (b^2 + b_\tau)/2 = -f^3 f_{tt}/2$ in the case of $\varphi(x, y, t) \sim b(x^2 + y^2)$ and $b(t) \neq t_0 - t$ does not equal zero. If we are interested in collapsing solutions with [27, 13] $f^2(t) \sim (t_0 - t)/\ln[\ln(t_0 - t)]$, the structure of the self-similar nonlinear core [13] of the solution is described by the solutions of the following equation

$$-\lambda\Phi = -\frac{1}{2}(\nabla - i\mathbf{A})^2\Phi + A_0\Phi - g|\Phi|^2\Phi\,. \tag{36}$$

In the next section, by numerical calculation, we find the zero-energy localized ground state of GNSE (36). We show the dependence of its effective width on the parameter $C = g|k|$ as well as the form of the functions u, v, w.

3. Solution structure

For the numerical analysis of the solutions of Eq. (36) we use the method of the stabilizing multiplier [28]. The iteration approach for Eq. (36), which differs from Eq. (19) by the additional term $-\lambda\Phi$ in the LHS has the form

$$\Phi_{n+1} = M_n F^{-1}\left(G(p)F\left(-2C\Phi_n^3 + j\Phi_n(u^2+v^2-w)_n\right)\right), \tag{37}$$

$$M_n = \left(\frac{\int d^2p(F\Phi_n)^2}{\int d^2pG(p)F\Phi_nF(-2C\Phi_n^3 + j\Phi_n(u^2+v^2-w)_n)}\right)^\alpha . \tag{38}$$

Here F (F^{-1}) are the operators of direct (inverse) Fourier transform, $G(p) = -\left(p^2+\lambda\right)^{-1}$. The multiplier $j=1$ or $j=0$ if we take into account the nonlinear contribution of gauge fields in Eq.(36) or neglect it. In the case $j=0$ the usual normalization in NSE corresponds to $C=1/2$. Without the restriction of generality we shall suppose below that $\lambda = 1$.

We should choose the exponent α in the stabilizing multiplier M_n comparing the degrees of homogeneity of terms in LHS and RHS of Eq. (36) proceeding from the requirements that $M_n \to 1$ at $n \to \infty$. Without the term $\Phi(u^2+v^2-w)$ the exponent α equals 3/2. New features of our problem are that the nonlinearity in Eq. (36) has the polynomial character of the type $-2C\Phi^3 + b\Phi^5$ because both of the terms Φw and $\Phi(u^2+v^2)$ are proportional to Φ^5. Therefore, for the convergence of the iteration approach α should belong to the range $5/4 \le \alpha \le 3/2$. In the simulation of the present paper we have used the value $\alpha = 3/2$ which gives rapidly the value $M_n = 1$ of the stabilizing multiplier. We have used the distribution of the form $\Phi(\zeta_x,\zeta_y) = \left(\sqrt{\gamma\delta}/\pi\right)\exp(-\gamma{\zeta_x}^2 - \delta{\zeta_y}^2)$ as the initial field configurations with arbitrary constants γ and δ. In our calculations we have obtained rapidly the isotropic solutions.

To regularize the integrals (6) and (7) which diverge logarithmically in coincident points at numerical calculations of gauge potentials u and v we substituted $r^2 \to r^2+\varepsilon^2$ in the expression for Green function (8). In the momentum space this corresponds to the substitution of $d^3p\, f(\varepsilon p)$ with $f(\varepsilon p) = \int\limits_0^\infty dm \exp(-\sqrt{m^2+\varepsilon^2p^2})$ for d^3p. The factor $f(\varepsilon p)$ at $\varepsilon p \gg 1$ decreases exponentially cutting off all momentum integrals. However, the infrared region remains the same, because at $\varepsilon p \ll 1$ $f(\varepsilon p) = 1$. In our calculations the cut-off radius ε which has the sense of the thickness of vortex core, was equal to 10^{-2}.

Simulation was performed on the square lattice with linear sizes $L_x = L_y = 12$. The maximum number of lattice sites was limited by the value $n = n_xn_y = 128 \cdot 128$. To test our approach we used the solution of EM (36) with $A_\mu = 0$ ($j=0$) and with $C = 1/2$ which gives the well-known value $N = 11.703$, as well as the solution of a self-dual equation $\Delta \ln\rho = -\rho^2$ [8], when $w = -\rho^2$, $u = \partial_y \ln\rho$, $v = -\partial_x \ln\rho$.

The Figures 1-3 show the configurations of the fields ρ, u and w for the specific value of the parameter $C = 4$. We may obtain the form of the function $v(\zeta_x, \zeta_y)$ using the relation $v(\zeta_x, \zeta_y) = -u(\zeta_y, \zeta_x)$.

Using the obtained function ρ we computed the dependencies of the critical power N (the particle number) and the effective width $< R^2 >= N^{-1} \int d^2\zeta\, \boldsymbol{\zeta}^2 \rho(\boldsymbol{\zeta})$ on the parameter C. The results of calculations are given in the Table 1.

Table 1.

j	C	N	$< R^2 >$
0	0.5	11.703	1.2607
1	3	2.9216	1.2464
1	5	1.2825	1.2579
1	10	0.5973	1.2600
1	100	$5.8528 \cdot 10^{-2}$	1.26066
1	1000	$5.8516 \cdot 10^{-3}$	1.26066

4. Conclusion

In this paper we have studied systematically the influence of the CS gauge field, reflecting the specific feature of the dimensionality of our problem, on field configurations in the GNSE model. Here we summarize some new results.

It is seen from Eqs. (15)-(19) that if we neglect CS gauge fields ($j = 0$ in Eq. (37)) the dependence of the particle number N on the parameter $C = g|k|$ has the form $N = N_0/C$. This dependence is shown in Fig.4 by the dotted line. It follows from the results shown in the first line of Table 1 that $N_0 = 5.585$. The contribution of CS gauge fields (j=1 in the Table 1) leads to the sharp decrease of values of N. In particular $N_{j=1}(3)/N_{j=0}(0.5) \approx 0.25$. The effective width $< R^2 >$ changes slightly. The calculated dependence $N(C)$ at $C \geq 3$ is given in Fig.4.

As it should be expected at a fixed value of the parameter C in the region $C \geq 3$, $N_{j=1}(C)$ is always greater than $N_{j=0}(C)$, because the CS gauge fields describe effective repulsion. In the range $1 < C < 3$ we could not perform calculations in the framework of the method used, due to the breaking of convergence of the iteration method (37), (38). We cannot explain this phenomenon using the theory presented in this paper. Note however, that the values $C = 1$ and $C = 2$ at $g = 1$ correspond to the discrete values $k = 1$ and $k = 2$ which describe the field configurations in the self-dual limit $k = 1$ [8] and the one-half fermion limit $k = 2$, which qualitatively differ from our case. The classic

limit of the considered theory corresponds to the case $k \to \infty$, when the gauge field splits off from the field $\Psi(x, y, t)$. For Eq. (19) and for the value $N(C)$ the limit $C \gg 1$ denotes that $N(C) \to 0$.

Strong Langmuir turbulence in plasmas is usually described by the solutions of the NSE (Eq. 24). It is assumed that a cascade of the randomly distributed self-similar collapsing fields is generated. In this paper we show that the specific features of spatial two-dimensional systems may lead to the traditional picture of turbulence associated with Euler equations. However, for the hydrodynamical mechanism of turbulence (HMT) to be involved, it is necessary the existence of a linear profile of the phase $\varphi(x, y)$ $(= ax + by)$ in each mode. This implies that the nonlinear contributions (by x and y) in the temporal evolution of the phase are small.

One of the media in which the HMT can play a role is an optical medium with random inhomogeneous guiding surfaces. Reflecting from the surfaces, wave fronts acquire random directions of propagation. For media with weak Kerr nonlinearity, the nonlinear phase disturbance from adjacent points will not be important. Application of the HMT model suggested above requires separate consideration and will be presented elsewhere.

In conclusion, we studied numerically the structure of the collapsing mode in GNSE model, observed the effect of strong reduction of the critical power N in spatial two-dimensional systems as compared to the traditional values, and showed that in the case of appropriate boundary conditions the phenomenon of collapse inhibits the development of turbulence according to the hydrodynamic scenario.

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Figure captions

FIG. 1. Plot of the function $\rho(\zeta_x, 0) = \rho(0, \zeta_y)$.

FIG. 2. Plot of the function $u(\zeta_x, 0)$.

FIG. 3. Plot of $w(\zeta_x, 0) = w(0, \zeta_y)$.

Fig. 4. Number of particles N as a function of the parameter C without taking into account the gauge field (dotted line) and with the gauge field (solid line). The point denotes the value $N(0.5) = 11.703$.